B.Sc. (Part-II) (CBCS Pattern) Semester-IV

USMT-07 - Mathematics Paper-I - Algebra

P. Pages : 2 **GUG/S/25/12014(S)**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Let G be the set of all 2x2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b, c, d are real numbers, such that $ad - bc - \neq 0$. For the operation in G use the multiplication of matrices. Show that G is an infinite non abelian group. **6**

b) Let a, b are in group G, then prove that the equations ax = b and ya = b have unique solutions for x and y in G. **6**

OR

c) Prove that $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8)^4 = (1\ 5)\ (2\ 6)\ (3\ 7)\ (4\ 8)$. **6**

d) Prove that a non – empty subset H of the group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$. **6**

UNIT – II

2. a) Let a $\in$ G be arbitrary and Let H be a subgroup of a group G, then prove that $Ha = H \Leftrightarrow a \in H$. **6**

b) Show that the intersection of two normal subgroups of G is a normal subgroup of G. **6**

OR

c) Let H be a subgroup of G. Then prove that there is a one-to-one correspondence between any two right cosets of H in G. **6**

d) Prove that if G is a finite group and H is a subgroup of G, then 0(H) is a divisor of 0(G). **6**

UNIT – III

3. a) Let G be any group, g be a fixed element in G. Define $\phi : G \to G$ by $\phi(x) = g x g^{-1}$. Prove that ϕ is an isomorphism of G onto G^1. **6**

b) Let N be a normal subgroup of G. Define the mapping $\phi : G \to G / N$ such that $\phi(x) = Nx, \forall x \in G$. Then prove that ϕ is a homomorphism of G onto G/N. **6**

OR

c) If ϕ is a homomorphism of G into G' with Kernel K, then prove that K is normal subgroup of G. **6**

d) Prove that any infinite cyclic group in isomorphic to the additive group of integers. **6**

UNIT – IV

4. a) Prove that a ring R is commutative if and only if $(a+b)^2 = a^2 + 2ab + b^2$. **6**

b) If R is a ring with zero element 0, then prove that for all $a, b \in R$. **6**

i) $a0 = 0a = a$

ii) $a(-b) = (-a)b = -(ab)$

iii) $(-a)(-b) = ab$

OR

c) Prove that the intersection of two subrings is a subring. **6**

d) Show that the ring $R = \left\{a + b\sqrt{2} \,/\, a,\ b \in z\right\}$ is an integral domain under addition and multiplication. **6**

5. Solve **any six.**

a) Prove that the identity of a group G is unique. **2**

b) If the group has two elements, show that it must be abelian. **2**

c) Define left coset and right cost of subgroup H in G. **2**

d) Define Normal subgroup. **2**

e) Define Group homomorphism. **2**

f) Find Kernel of a homomorphism $\phi : G \to G$ such that $\phi(x) = 13x,\ \forall x \in G$. **2**

g) Define ring. **2**

h) Define Integral domain. **2**

****